IMAGE REGISTRATION TECHNIQUES WITH MULTIRESOLUTION ANALYSIS IN SATELLITE OCEANOGRAPHY

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ABSTRACT

In this paper we present processing techniques for automated image-to-image geometrical registration . One reference image is used to register the working image. Three methods are used. The first method is the classical image registration method using the *maximum cross-correlation* (MCC) in the spatial domain [1]. The second method is based on MCC and multiscale analysis through wavelet multiresolution techniques [2]. The third one is a fusion of the two previous methods. For each technique the transformation coefficients relating both images are estimated. Finally the second image is transformed and georeferenced to the first one. In the conclusion, a proposal of quantitative parameters leads to a final discussion on the results.

RESUM

En aquest article es fa una descripció dels procediments realitzats per enregistrar dues imatges geomètricament, de forma automàtica, si es pren la primera com a imatge de referència. Es comparen els resultats obtinguts mitjançant tres mètodes. El primer mètode és el d'enregistrament clàssic en domini espacial maximitzant la correlació creuada (MCC) [1]. El segon mètode es basa en aplicar l'enregistrament MCC conjuntament amb un anàlisi multiescala a partir de transformades wavelet [2]. El tercer mètode és una variant de l'anterior que es situa a mig camí dels dos. Per cada un dels mètodes s'obté una estimació dels coeficients de la transformació que relaciona les dues imatges. A continuació es transforma per cada cas la segona imatge i es georeferencia respecte la primera. I per acabar es proposen unes mesures quantitatives que permeten discutir i comparar els resultats obtinguts amb cada mètode.

RESUMEN

En este artículo se describen los procedimientos realizados para registrar geométricamente dos imágenes de forma automática si se toma la primera como imagen de referencia. Se comparan los resultados obtenidos mediante tres métodos. El primero es el método clásico para registrar dos imágenes en el dominio espacial maximizando la correlación cruzada (MCC) [1]. En el segundo se trata de aplicar de forma conjunta técnicas de análisis multiescala, basadas en la transformaciones wavelet y el método MCC [2]. El tercero es una variación del segundo situada a medio camino de los dos métodos anteriores. Para cada método se obtiene una estimación de los coeficientes de la transformación que relaciona las dos imágenes. A continuación se transforma la segunda imagen que se georeferencia respecto a la primera para cada caso. Para finalizar se proponen unas medidas cuantitativas que nos permiten discutir y comparar los resultados obtenidos en cada uno de los métodos.

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1 INTRODUCTION AND OBJECTIVES

Our framework is the Ph.D. proposal development: *Aplicació de tècniques de fusió de dades per a l'anàlisi d'imatges de satèl·lit en Oceanografia* - *Data fusion techniques for application on satellite Oceanography image analysis -,* Ramon Reig DET-EPS-UV, co-directed by Dr. Vicenç Parisi DEE-UPC and Dr. Emili García Ladona GOF-CMIMA-CSIC. The main reference is the Ph.D. work [4]: *Analysis of Mesoscale Structure through Digital Image Processing Techniques,* Dr. Vicenç Parisi, co-directed by Dr. Emilio García ICM-CSIC and Dr. Joan Cabestany DEE-UPC.

From a general point of view our present research effort is focused on studying structures and phenomenon present in oceanographic satellite images by means of digital image processing techniques. Especially through image-fusion methods [5], using images from different sensors and/or satellites. With these techniques we need pre-processing steps in order to obtain meaningful results. One of these steps is geometrical registration.

In this paper we describe automated image-to-image geometrical registration methods. The first image is used as a reference. This pre-processing assures proper image fusion and superimposition onto the same ground truth, being aware that these images belong to different sensors or satellites. Ground Control Points (GCP) are needed in each image to carry out any registration process. In this case the possible GCP are coastline points, because they are static reference points, and they are present in both images.

As a working image we use a SST (Sea Surface Temperature) image from the Alboran Sea (38ºN-6ºW, 35ºN-2ºE) the western basin of the Mediterranean Sea. There is a large current that runs to and from the Atlantic Ocean through the Strait of Gibraltar. The darker zones represent colder waters from the Strait jet. Using a pseudocolor zebra palette, the gray-level image is optimized for visualization purposes, as we see in *figure 2.1*. With this palette, the level changes are reinforced and the structures in the image are easily seen.

We discuss three registration techniques :

1. Image registration with classical Maximum Cross-Correlation (MCC) in the spatial domain [1].
2. Image registration MCC with multiscale analysis, 1st version [2],[3].
3. Image registration MCC with multiscale analysis, 2nd version.

For each method, an estimation of the transformation coefficients relating both images is obtained. There is also a proposal of comparative parameters in order to quantify the coefficients suitability. The transformation coefficients are used to de-transform the second image referenced to the first one. The next step is to show the de-transformation on a contour image and on a gray-level image . For comparative purposes two difference images are also defined and shown. Finally comparative measures are proposed and the results are discussed.

2 FUNDAMENTALS AND METHODS

The reference image is an SST image, **Image_1.** The second image is obtained from a rotation **salfaº (10º)** and a horizontal shift **sdh (5 pixels), Image_2**. This artificially modified image is used in order to quantify the quality of the registration methods (*figure 2.1)*. We also need other parameters: analysis window size **sdima (9x9)**, search window size **sdimc (40x40)**, scale change factor **sfact (2)** and last scale value **sL (2)**.

To register **Image_2** to **Image_1** we will use their coastline points. The corresponding coastline contour images are obtained and shown (*figure 2.2)*. All the necessary steps used during any registration process are described in the correspondent sections.

For each registration method there is a selection of GCP pairs in section 2.1. The next step is the estimation of the transformation coefficients relating GCP pairs in both images, in section 2.2. In addition, the corresponding results table has been filled out. A set of error parameters is proposed to assist the choice of coefficients in the multiscale cases. In section 3 the transformation coefficients are used to de-transform the second image referenced to the first one. Showing the de-transformation on a contour image and on a gray-level image is the next step. For comparative purposes two difference images are also defined and shown in section 4.

Finally comparative measures are proposed and the results are discussed.

The proposed methods are more computer consuming than classical MCC, basically due to the wavelet analysis. But in our case the wavelet analysis will be used anyway when you apply the image-fusion techniques.

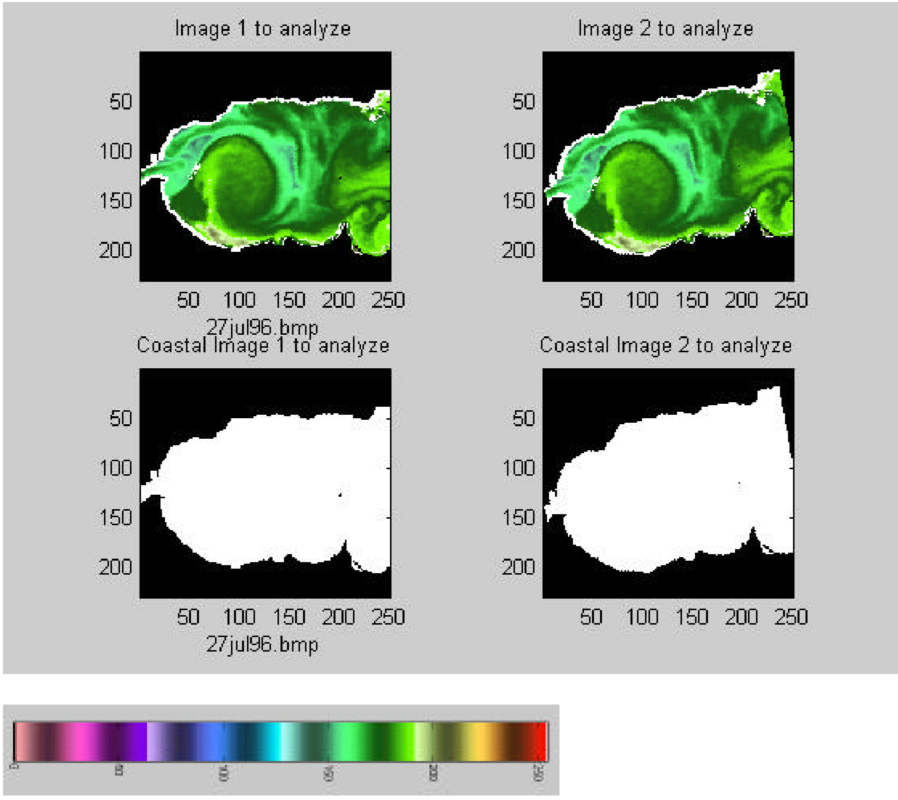


***figure 2.1 Images 1 & 2**, and corresponding* coastline *images. Zebra palette.*

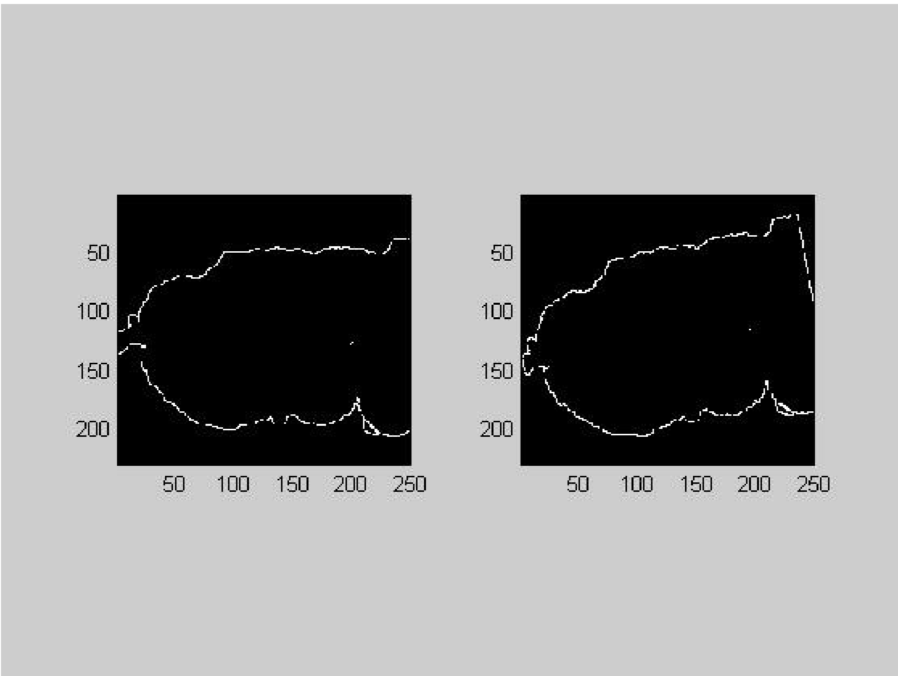


figure 2.2** Coastline contour image **Image_2** & **Image_1

2.1 GCP pairs selection with three methods

Following below there is a description of GCP selection and the one-to-one GCP matching process for each method: 2.1.1 MCC in spatial domain; 2.1.2 MCC & multiscale analysis; 2.1.3 MCC & multiscale analysis, 2nd version.

2.1.1 Maximum Cross-Correlation (MCC) in the spatial domain

Historically this is one of the first registration methods [1]. It is based on matching ground control points (GCP) from two images in the spatial domain. In this particular case, there is a slight variation with respect to the classical method [1]; the ground control points can *only* be coastline points. This assumption speeds up the matching points search.

We are looking for matching points between the coastline of **Image_2** and the coastline of **Image_1**. The matching criterion is the maximizing cross correlation (MCC) of analysis windows centered on the corresponding points.

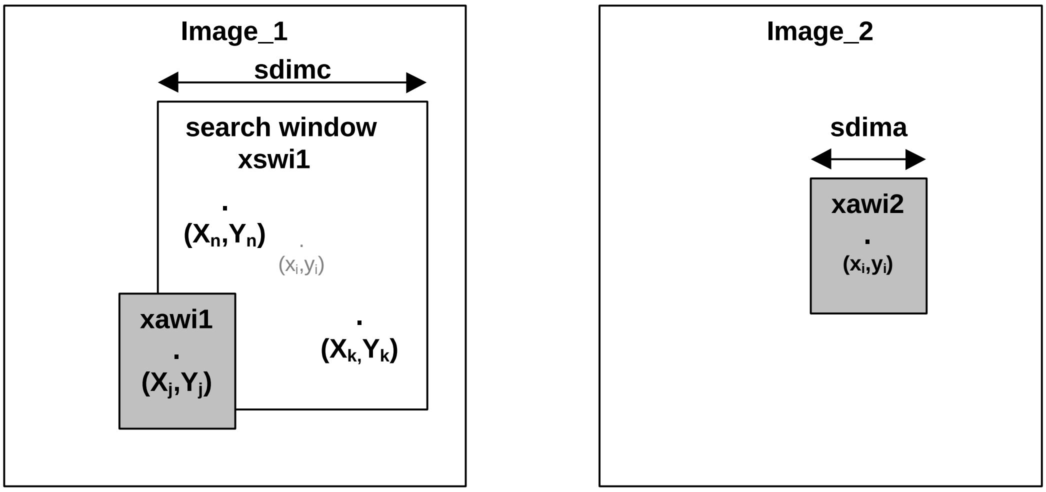


figure 2.3 *Matching coastline points of* ***Image_2*** *&* ***Image_1***

For every single coastline point in **Image_2** (x_i, y_i):
a.- An analysis window with size **sdima** (9X9) centered on the point (x_i, y_i) is defined in **Image_2, xawi2.**
b.- All coastline points inside a **sdimc** (40x40) search window, **xswi1** in **Image_1,** centered on the same value (x_i, y_i) are identified.
c.- For every candidate (in *figure 2.3* there are three) a new analysis window is defined centered on the corresponding point, **xawi1** – in *figure 2.3* it is centered on the (X_j, Y_j) point-.
d.- The *normalized correlation* of the two windows **xawi1 & xawi2** is calculated as:

$$norm\_corr = \frac{\sum\sum \mathbf{xawi1} .* \mathbf{xawi2}}{\left[\sum\sum \mathbf{xawi1}^2\right]^{1/2} \bullet \left[\sum\sum \mathbf{xawi2}^2\right]^{1/2}} \quad (1)$$

e.- From the candidate points in **Image_1** we select the one with a maximum on the *normalized correlation* .
f.- If there are more than one point in **Image_1** with the same maximum value, the pairing of the corresponding point from **Image_2** is rejected.
g.- Go back to the first step for the next coastline point of **Image_2.**

There are special cases when different points on **Image_2** are matched with a single point on **Image_1**. A function is then used on the matching points list to solve the problem. A one to one matching is forced, leaving only the pairing with the highest cross-correlation value from the candidate pairs.

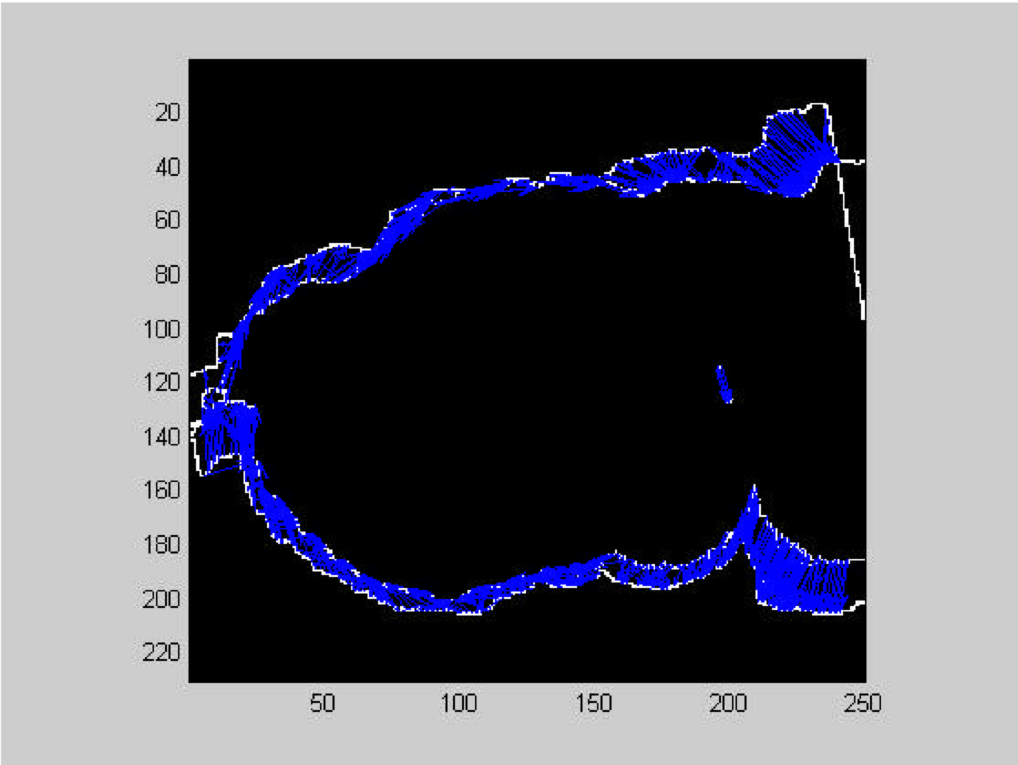


figure 2.4 *Matching coastline points of **Image_2** and **Image_1***

2.1.2 MCC with multiscale analysis, 1st version

Method [2], [3] is based on the application of a recursive wavelet transform until scale **sL** with an '*à trous'* algorithm on two images. For each pair of corresponding scale transforms we get a set of significant points on both images, named *Ground Control Points (GCP)*. They represent characteristic points of the images. Afterwards we look for GCP matching with maximum cross correlation criterion.

The wavelet decomposition allows the separation of image objects belonging to different spatial scales. The '*À trous'* algorithm (with holes) assures the projection of original images into consecutive nested subspaces on successive dyadic scales (2, 4, 8...), through recursive low-pass filtering approximate images are obtained at each scale, from fine to coarse resolution. It is equivalent to applying a wavelet analysis with cubic B-spline basis function [4].

There are some differences with the method [2], [3], described in *figure 2.5:*
1.- GCP selection is not based on maxima criterion. The selection is made based on several conditions. The first one is that candidate points must be coastline points.
2.- There is also a regional condition. GCP must be distributed on **snmaxr** regions (i.e. 4 regions).
3.- The final condition is that GCP must be singular points in their region.

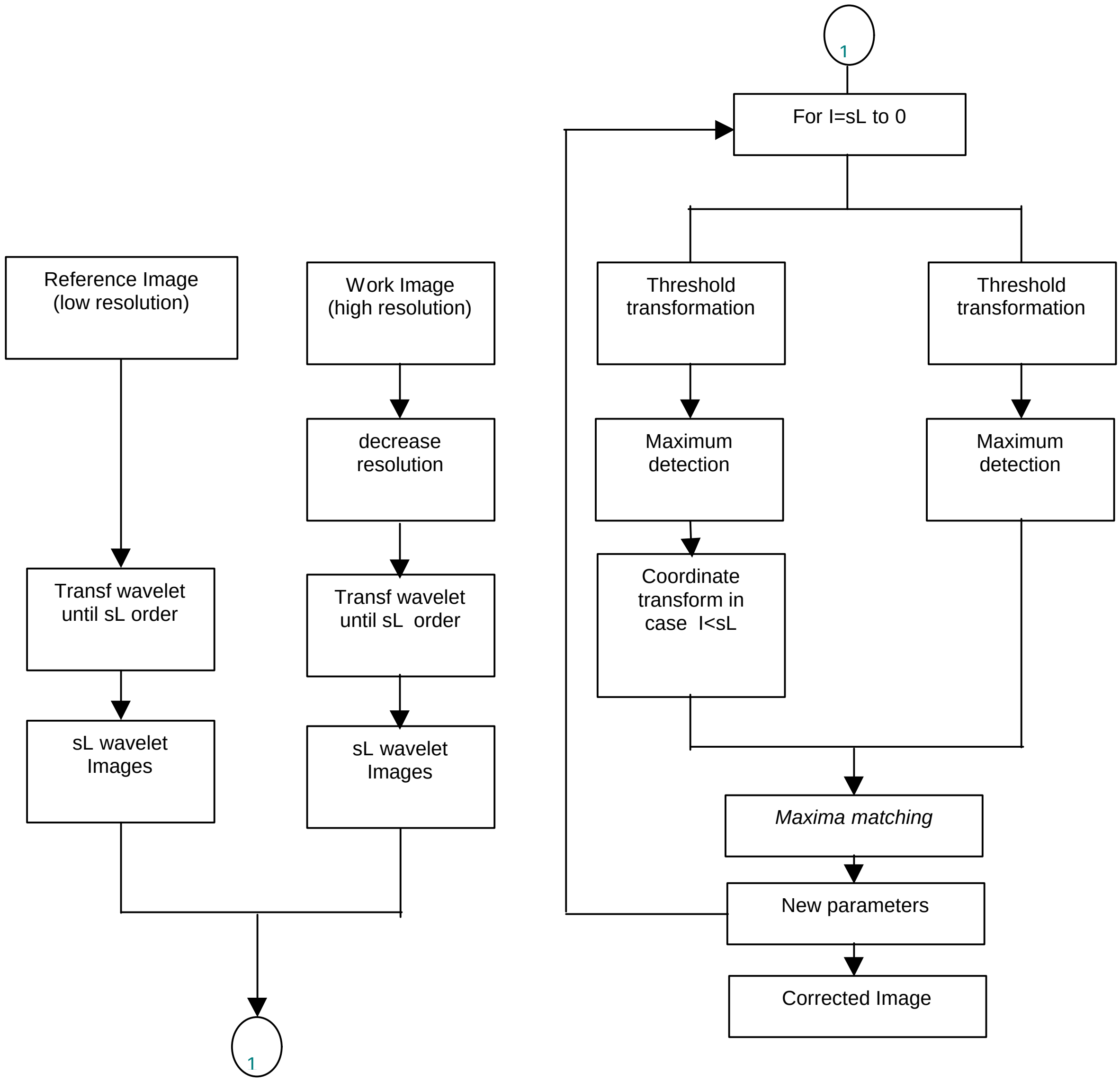


figure 2.5 *Flow diagram for registration method based on multiscale analysis and MCC*

Wavelet transform with '*à trous*'

This multiresolution analysis consists in the approximation of an image in different nested subspaces V_i, V_{i+1}, V_{i+2} ... embedded into each other: $V_{i+1} \subset V_i$. The change from one subspace to another is the result of a scale change (*figure 2.6).* With the *'à trous'* algorithm, the original image *f(x,y)* can be projected onto a subspace V_0 by means of a scalar product with a scaling function $j_0(x,y)$. The result is $c_0[m,n]$ *{m,n Î Z}:*

$$c_0[m,n] = \left\langle f(x,y), j_0(x-m, y-n) \right\rangle$$

The approximation at successively twice magnified scales can be obtained by:

$$c_j[m,n] = \left\langle f(x,y), \frac{1}{4^j} j_0\left(\frac{x-m}{2^j}, \frac{y-n}{2^j}\right) \right\rangle$$

with

$$j_j(x,y) = \frac{1}{4^j} j_0\left(\frac{x}{2^j}, \frac{y}{2^j}\right)$$

The details at each scale *$w_j[m,n] = c_{j-1}[m,n] - c_j[m,n]$*

And the scaling function at each scale can be expressed as the output of a 2D filter:

$$\frac{1}{4^{j+1}} j_0\left(\frac{x}{2^{j+1}}, \frac{y}{2^{j+1}}\right) = \sum_{k,l \in Z} h[k,l] j_0\left(\frac{x}{2^j} - k, \frac{y}{2^j} - l\right)$$

which permits an approximation at successive scales to be obtained in a recursive way:

$$c_{j+1}[m,n] = \sum_{k,l \in Z} h[k,l] c_j\left[2^j k + m, 2^j l + n\right]$$

As the scaling function we use the cubic B-spline:

$$j(r) = \frac{3}{8}\left(\frac{sin\frac{pr}{4}}{\frac{pr}{4}}\right)^4$$

$$r = \sqrt{x^2 + y^2}$$

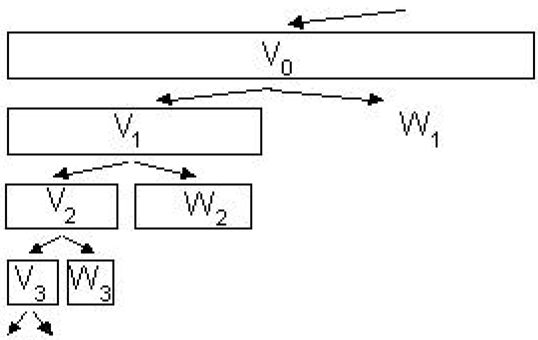


figure 2.6 *Approximation subspaces using a multiresolution decomposition scheme. The length of each rectangle symbolizes the resolution , from fine to coarse.*

GCP selection from scale sL down to 0

In each scale, GCP are selected from coastline points in the images. Transform images in the corresponding scales are used in the next steps (scale 0 are original images):

a.- Every GCP candidate is parameterized with a mean gray-level value from an analysis window around the point in the corresponding transform image.

b.- The image is divided in **snmaxr** (i.e 4) rectangular regions. The regional condition imposes the selection of points in every region if there is more than one possibility (more than one coastline point).

c.- In every region a set of GCP are selected. They are *singular points* of the region. That means they have a *singular value* in the selected parameter. A *singular value* is a value with a

minimum in the parameter probability density function (pdf) approximation.
The selection of *singular points* in each region is carried out by:

c.1.- Coastline points parameter histogram (*figure 2.7, 2.8)*.

c.2.- GCP are **sper** percent of the region points. The first group of GCP are all the points with a mean gray-value of the analysis window that is a minimum in the corresponding histogram. The remaining GCP are obtained with the consecutive values from the minimum upwards.

d.- The union of the results in every region gives the final GCP from both images in a particular scale.

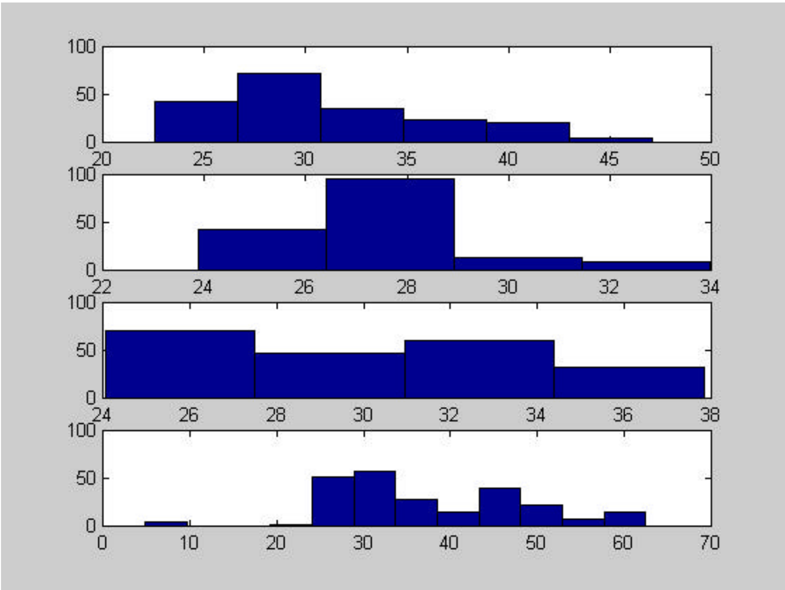


figure 2.7 *Mean gray-level histograms for every region of* ***Image_1*** *scale 1*

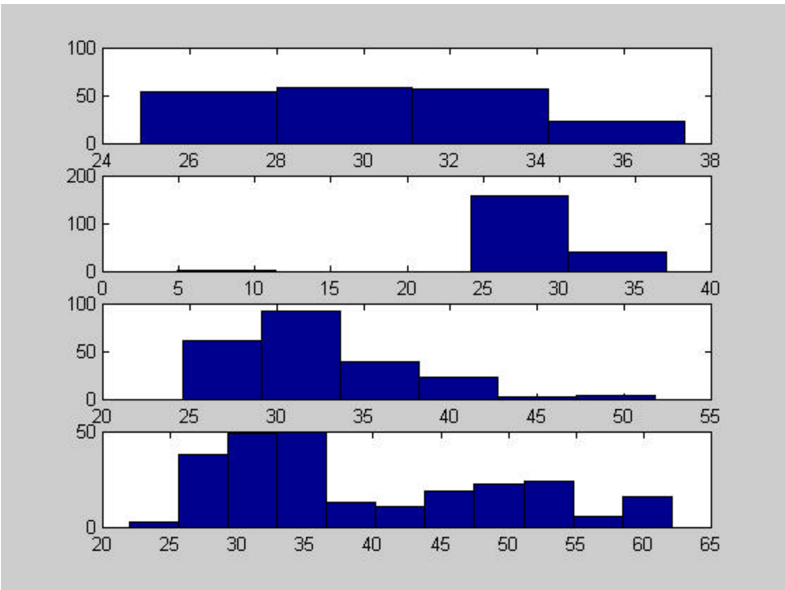


figure 2.8 *Mean gray-level histograms for every region of* ***Image_2*** *scale 1*

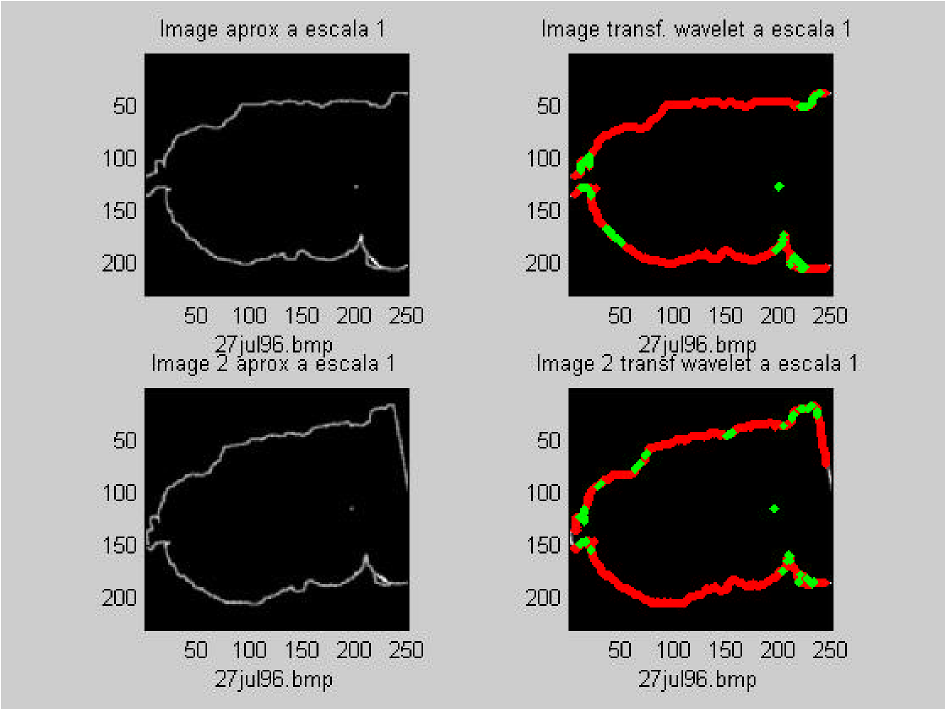


figure 2.9 *Wavelet transform at scale 1 from Images 1&2 , with the corresponding approximation images. On transform images, contour points are superimposed (red points), and the selected GCP are the green points*

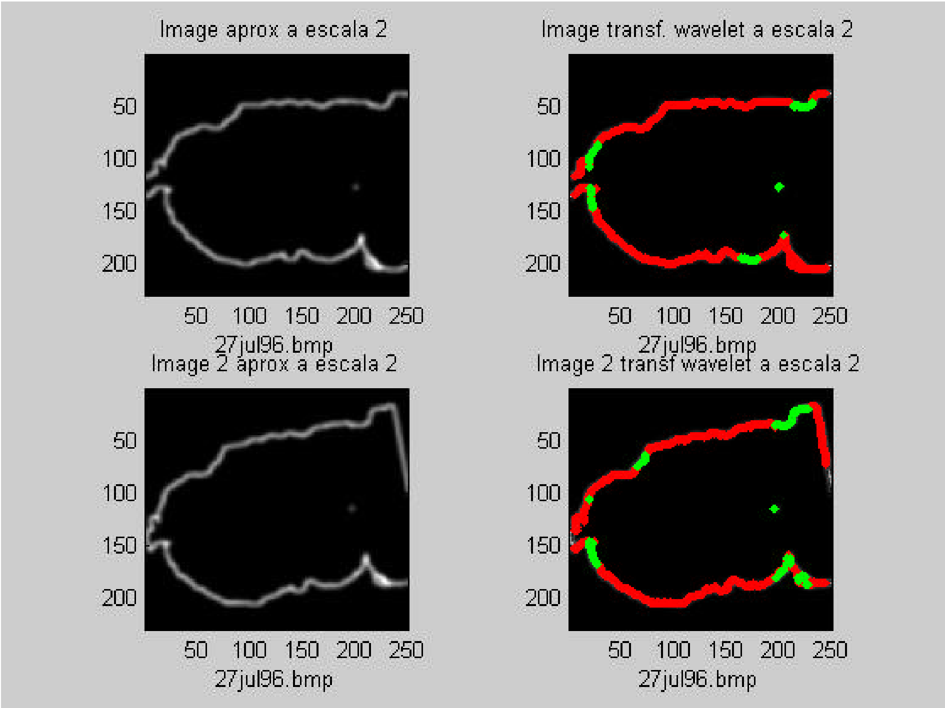


figure 2.10 *Wavelet transform at scale 2 from Images 1&2, with the corresponding approximation images. On transform images, contour points are superimposed (red points), and the selected GCP are the green points*

GCP matching with Maximum Cross-Correlation criterion

GCP in **Image_2** are matched to the GCP in **Image_1**. The matching criterion is *Maximize Cross Correlation* (MCC), the same used in 2.1.1. A posteriori refinement forces one to one matching. Finally the pairings are shown in each scale.

GCP matching is applied at each scale and to each GCP in **Image_2***:*
a.- An analysis window **xawi2** centered on the point, is defined. With size depending on working scale, **sdim=sdima*(sfact)**^{si}, scale **si** (sdim = 9x9, 18x18, 36x36...).
b.- Inside a search window **xswi1** in **Image_1** with a size **sdimc** (40x40), we look for a candidate GCP corresponding pair.
c.- For each candidate an analysis window is defined **xawi1,** and the normalized cross-correlation from corresponding analysis windows is calculated.

$$norm\_corr = \frac{\sum\sum \mathbf{xawi1} .* \mathbf{xawi2}}{\left[\sum\sum \mathbf{xawi1}^2\right]^{1/2} \bullet \left[\sum\sum \mathbf{xawi2}^2\right]^{1/2}}$$

d.- From the candidate points in **Image_1** we will select the one with a maximum on the *normalized correlation* .
e.- If there is more than one point in **Image_1** with the same maximum value, the pairing with the corresponding point from **Image_2** is rejected.
f.- Go back to the first step with the next GCP in **Image_2**. When all the GCP are finished, the next scale is analyzed.

There are special cases when different points in **Image_2** are matched with a single point in **Image_1**. A function is used on the matching points list to solve the problem. A one to one matching is forced leaving only the pairing with the highest cross-correlation value from the candidate pairs.

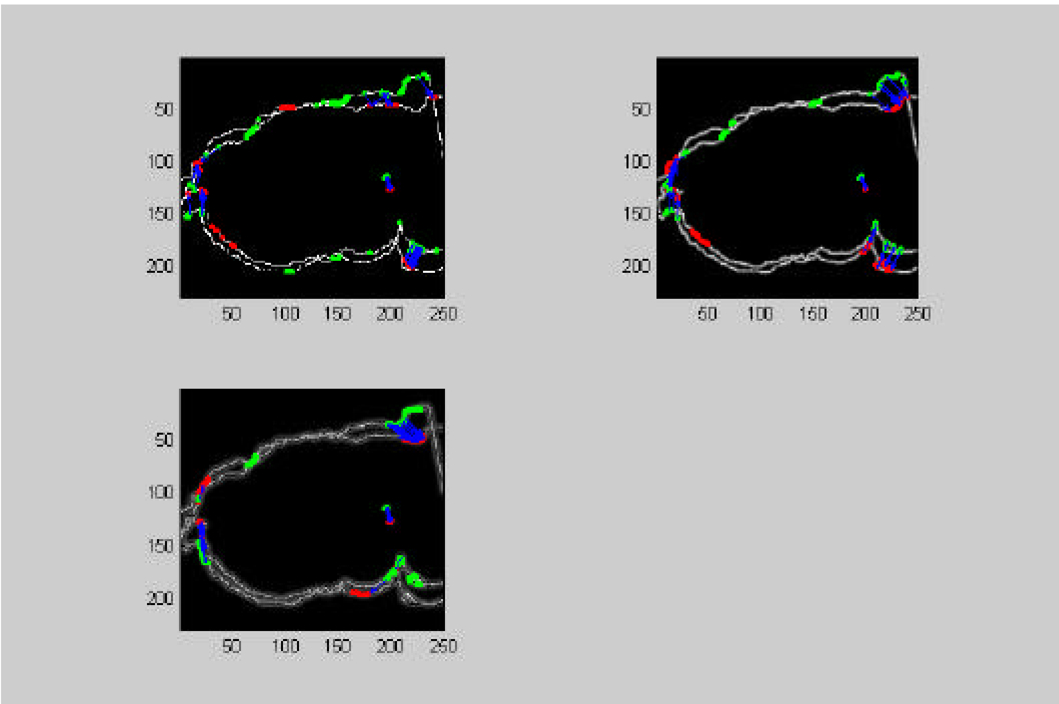


figure 2.11 *GCP matching from Image_2 (green points) & Image_1 (red points). Scales 0, 1 and 2*

2.1.3 MCC with multiscale analysis, 2nd version

This method is a variation on the last one. It can be seen as a fusion of the former methods. GCP are selected from coastline points only in **Image_2**, and in **Image_1** the GCP are all the coastline points. For every GCP in **Image_2** we look for a matching point in **Image_1** within a search window, maximizing cross-correlation.

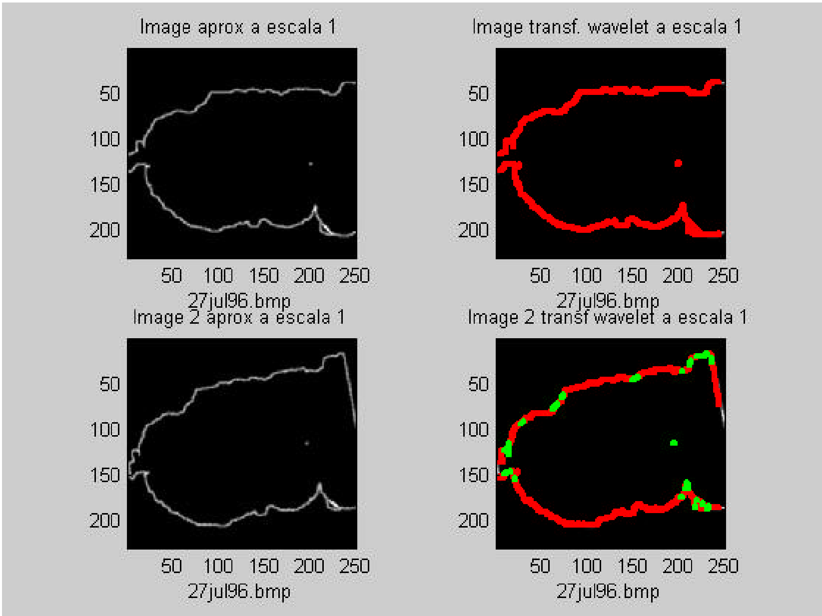


figure 2.12 ***Image_2*** *&* ***Image_1*** *at scale 1. Selected GCP in* ***Image_2*** *(green points). All coastline points in* ***Image_1*** *are GCP*

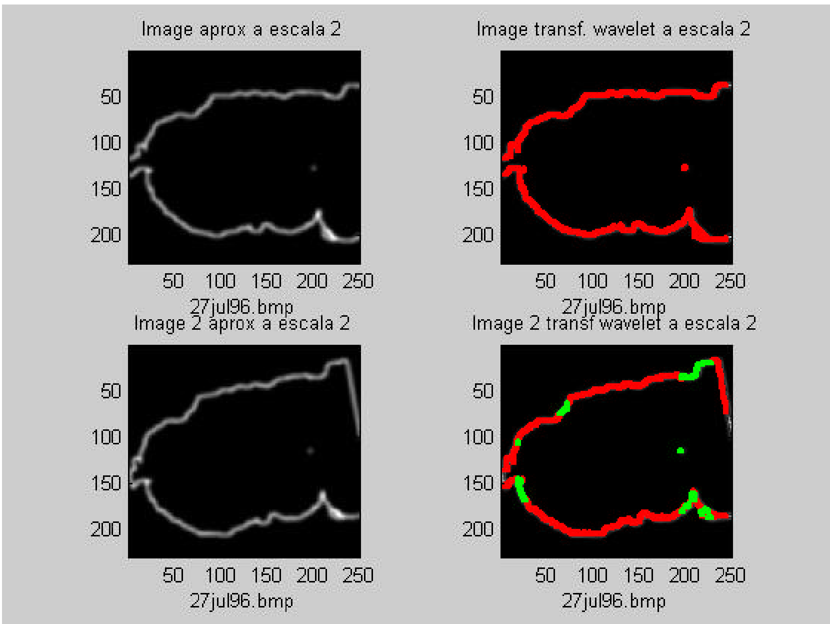


figure 2.13 ***Image_2*** *&* ***Image_1*** *at scale 2. Selected GCP in* ***Image_2*** *(green points). All coastline points in* ***Image_1*** *are GCP*

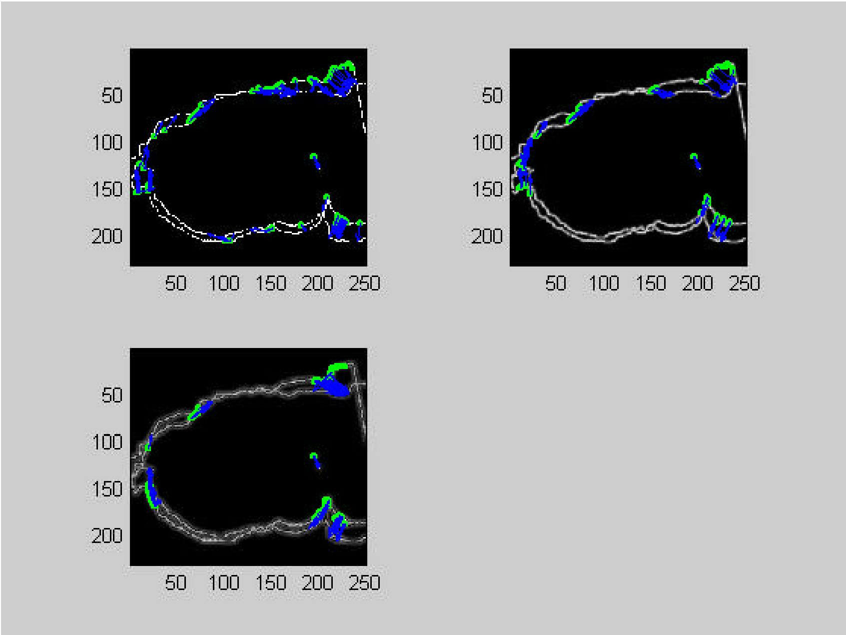
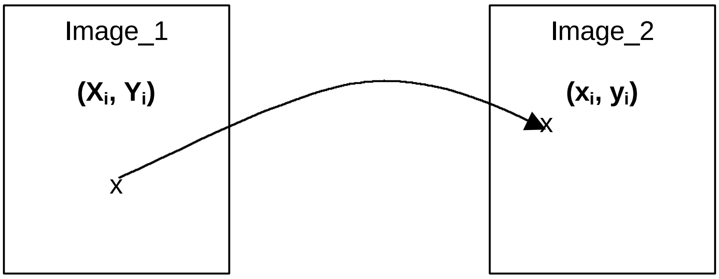


__figure 2.14__ GCP matching from Image_2 (green points) & Image_1. Scales 0, 1 and 2

2.2 Transformation coefficients relating pairs of GCP

Once we get N pairs of points - one from each image in every pair, with any of the preceding methods - we can approximate the value of the transformation coefficients relating both images.

We have two images and Image_1 is the reference one.



Direct transformation with the equation :

$$x_i = a_0 + a_1 \cdot X_i + a_2 \cdot Y_i$$
$$y_i = b_0 + b_1 \cdot X_i + b_2 \cdot Y_i \qquad (2)$$

could solve all basic transformations in the images (rotations, shifts, scaling). From the N pairs of points obtained, we define an equation system. We solve it by minimizing LSQ error (Least Squares sense).

$$\begin{bmatrix} 1 & X_1 & Y_1 \\ 1 & X_2 & Y_2 \\ . & . & . \\ 1 & X_n & Y_n \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ . \\ x_n \end{bmatrix} \qquad \begin{bmatrix} 1 & X_1 & Y_1 \\ 1 & X_2 & Y_2 \\ . & . & . \\ 1 & X_n & Y_n \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ . \\ y_n \end{bmatrix} \qquad (3)$$

$$\mathbf{MXY} \cdot \mathbf{a} = \mathbf{x} \qquad \mathbf{MXY} \cdot \mathbf{b} = \mathbf{y}$$

$$\mathbf{a} = \left(\mathbf{MXY}^{\mathbf{T}} \cdot \mathbf{MXY}\right)^{-1} \cdot \left(\mathbf{MXY}^{\mathbf{T}} \cdot \mathbf{x}\right) \qquad \mathbf{b} = \left(\mathbf{MXY}^{\mathbf{T}} \cdot \mathbf{MXY}\right)^{-1} \cdot \left(\mathbf{MXY}^{\mathbf{T}} \cdot \mathbf{y}\right) \qquad (4)$$

When we get the resulting coefficients, we propose a set of comparative parameters:

1. Mean absolute error on x component.
2. Standard deviation of absolute error on x component.
3. Mean absolute error on y component.
4. Standard deviation of absolute error on y component.
5. Dispersion Ratio, measures the degree of dispersion of the points in the image (better if closer to 1, near to 0 is the worst case).
6. Number of pairs used in calculations.

Dispersion ratio (**DR**) is defined as: mean value of Euclidean distances between the different points, normalized to the maximum possible distance of the corresponding images.

$$\mathbf{DR} = \frac{\sum_{\mathbf{l}} \mathbf{d_{ij}} / \mathbf{l}}{\mathbf{d_{max}}}$$

DR Dispersion Ratio

($0 \leq$ DR ≤ 1...)

$\mathbf{d_{ij}}$ Euclidean distance between points i,j
l number of different pairs of points
$\mathbf{d_{max}}$ maximum distance

MCC Classic:

coef		coef	
a_0	16.1997	b_0	-13.1971
a_1	0.9943	b_1	0,0872
a_2	-0.1365	b_2	1.0034
m_x	3.7624	m_y	6.6451
σ_x	4.3017	σ_y	5.2475
DR (Dispersion Ratio)	0.36		
number of points	657		

MCC with multiscale analysis, version 1:

coef	scale 2	scale 1	scale 0	coef	scale 2	scale 1	scale 0
a_0	21.3960	20.8532	20.5896	b_0	-24.8197	-19.7317	-7.4873
a_1	0.9971	0.9844	0.9942	b_1	0.1948	0.1534	0.0602
a_2	-0.1627	-0.1578	-0.1635	b_2	0.9840	0.9848	1.0066
m_x	1.0866	2.3501	1.8705	m_y	1.0042	2.0666	3.8055
σ_x	1.2000	4.4274	4.4654	σ_y	0.9937	3.6779	4.9036
DR	0.35	0.40	0.38				
n. points	49	48	43				

MCC with multiscale analysis, version 2:

coef	scale 2	scale 1	scale 0	coef	scale 2	scale 1	scale 0
a_0	23.8310	24.4495	21.6016	b_0	-22.8346	-22.3885	-17.8964
a_1	0.9758	0.9796	0.9849	b_1	0.1608	0.1686	0.1306
a_2	-0.1658	-0.1704	-0.1586	b_2	0.9937	0.9844	0.9911
m_x	1.8264	0.7132	2.1139	m_y	2.7116	1.3022	4.8000
σ_x	2.2382	0.8640	3.8403	σ_y	4.9294	2.6656	5.3746
DR	0.37	0.37	0.37				
n. points	103	80	109				

In each method the coefficients minimizing error values are selected. They are emphasized. The 2nd version of the multiscale method seems to give the better results. In this case, error parameters decrease or are the same as the other methods, and the coefficients seem better, as DR and the number of points increases.

Below are two other cases of MCC with multiscale analysis are shown for comparative purposes. The first one is a case without any GCP selection.

MCC with multiscale analysis, using all the GCP:

coef	scale 2	scale 1	scale 0	coef	scale 2	scale 1	scale 0
a_0	19.8749	21.0720	16.1997	b_0	-16.3572	-16.7069	-13.1971
a_1	0.9834	0.9797	0.9943	b_1	0.0928	0.1000	0.0872
a_2	-0.1491	-0.1527	-0.1365	b_2	1.0190	1.0105	1.0034
m_x	2.4185	2.3511	3.7624	m_y	5.5187	5.2412	6.6451
σ_x	2.1941	2.2874	4.3017	σ_y	5.4160	4.7476	5.2475
DR	0.35	0.36	0.36				
n. points	458	501	657				

There is an improvement with respect to the classical MCC (corresponding with scale 0). This can be seen as multiscale analysis improves MCC method.

The second case is a multiscale analysis with a GCP selection with a regional criterion but with a random selection of points inside a region.

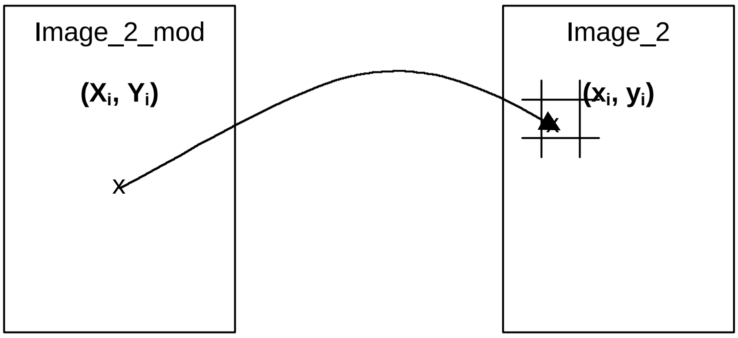
MCC with multiscale analysis with random GCP selection:

coef	scale 2	scale 1	scale 0	coef	scale 2	scale 1	scale 0
a_0	19.1935	22.0202	15.7850	b_0	-15.6127	-15.9110	-14.7885
a_1	0.9822	0.9737	0.9817	b_1	0.0847	0.0885	0.0713
a_2	-0.1445	-0.1510	-0.1162	b_2	1.0213	1.0205	1.0308
m_x	2.3168	2.6209	5.4584	m_y	5.6364	5.9919	7.5677
σ_x	2.0379	2.4182	5.3829	σ_y	5.0839	5.4417	5.5840
DR	0.35	0.36	0.37				
n. points	99	118	130				

Error parameters are higher, with great variation depending on the selected GCP. This seems to mean that regional criterion is important, but singular point selection is more important when selecting GCP. This first conclusion will be contrasted in sections 3 and 4.

2.3 Image to image registration

Using the **Image_2** and the coefficients obtained in 2.2 for each method, the image-to-image registration is made, and then we obtain **Image_2_modified.** This can be done with any pair of related images, either coastline images or complete SST images. **Image_1** will be used for comparative purposes.



For every single pixel of **Image_2_mod** we can get the corresponding point in **Image_2**, from transformation coefficients and equations (2). Normally the result is not an exact pixel position (it isn't integer), we use the *nearest neighbour pixel value* as an approximation. The corresponding pixel value is then mapped to the **Image_2_mod** original position.

Bellow there are same examples of registered images which are compared with ideal de-transformation. In each case we will do the registration on contour coastline images and also on complete images .

3 RESULTS AND COMPARATIVE MEASURES

Imatge_2_mod is obtained registering **Imatge_2** to **Imatge_1**. And **Imatge_resta=|Imatge_1 - Imatge_2_mod|** is the image of the difference between the ideal result and practical de-transformation. Results are shown for both: Coastline Images (*Figures 3.1, 3.3, 3.5, 3.7*) and complete SST Images (*Figures 3.2, 3.4, 3.6, 3.8*).

We obtain results tables with a set of parameters from difference images, in order to have quantitative comparative measures. A first set of parameters is obtained from difference images of coastline images. And a second set is obtained from the difference of complete images.

There are result tables for two other variants: multiscale without GCP selection, and multiscale with random GCP selection, where we can see that GCP selection is a critical aspect for multiscale cases

Classic MCC:

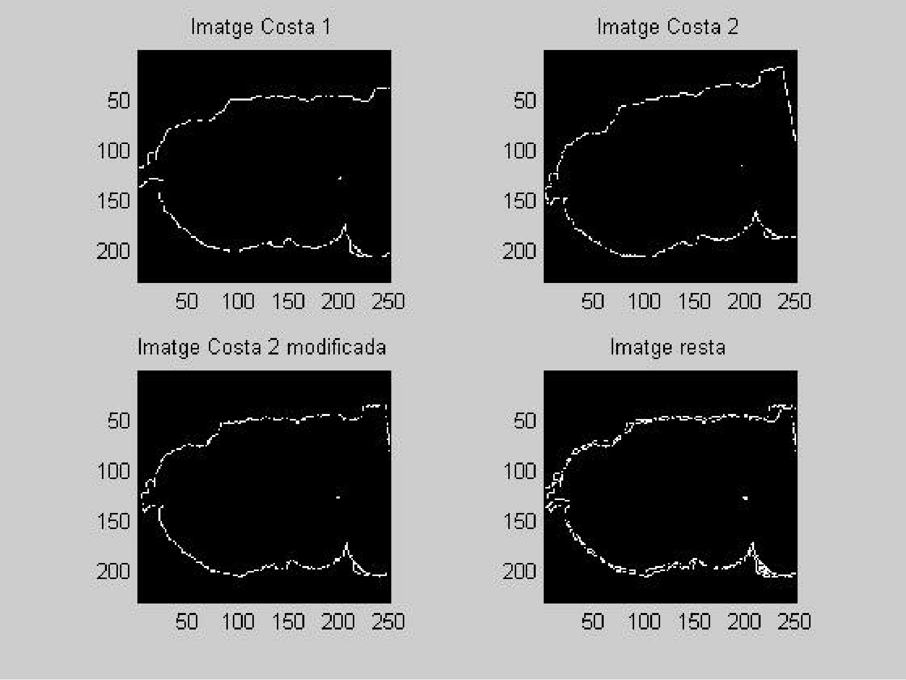


figure 3.1** Results of classical MCC. Registration on coastline images, from **Image_2** to **Image_2_mod. Image_1** is the ideal transformation result. The difference image is **|Image_1 – Image_2_mod|

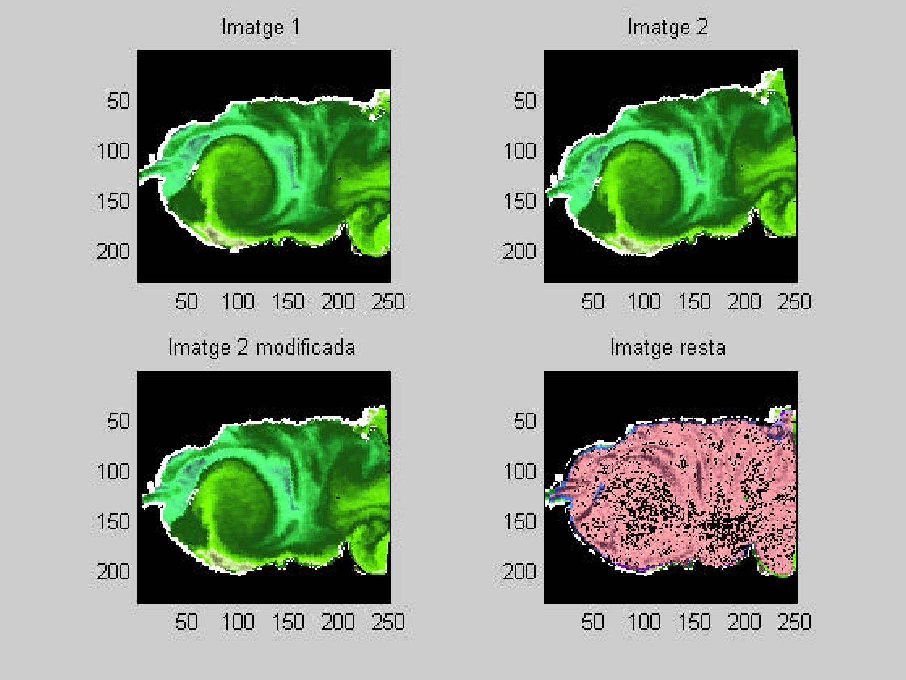


figure 3.2** Results of classical MCC. Registration on complete images, **Image_2_mod** is obtained from **Image_2. Image_1** is the ideal transformation result. The difference image is defined as **|Image_1 – Image_2_mod|

MCC with multiscale analysis, version 1

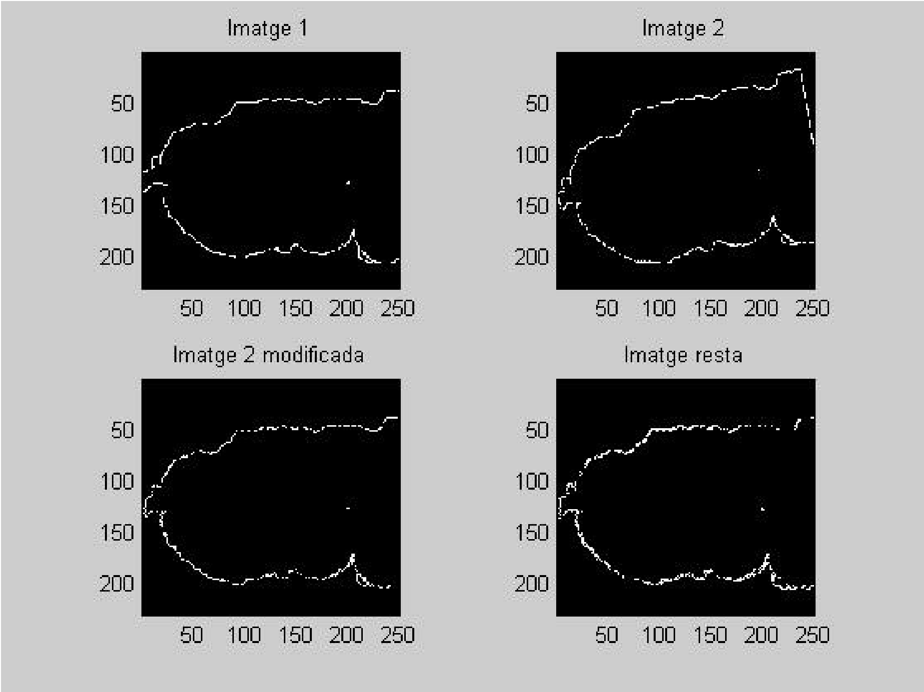


figure 3.3 *MCC with multiscale analysis, version 1. Registration of coastline images.* ***Image_2_mod*** *from* ***Image_2 Image_1*** *is the ideal transformation result. The difference image is defined as* ***|Image_1 – Image_2_mod|***

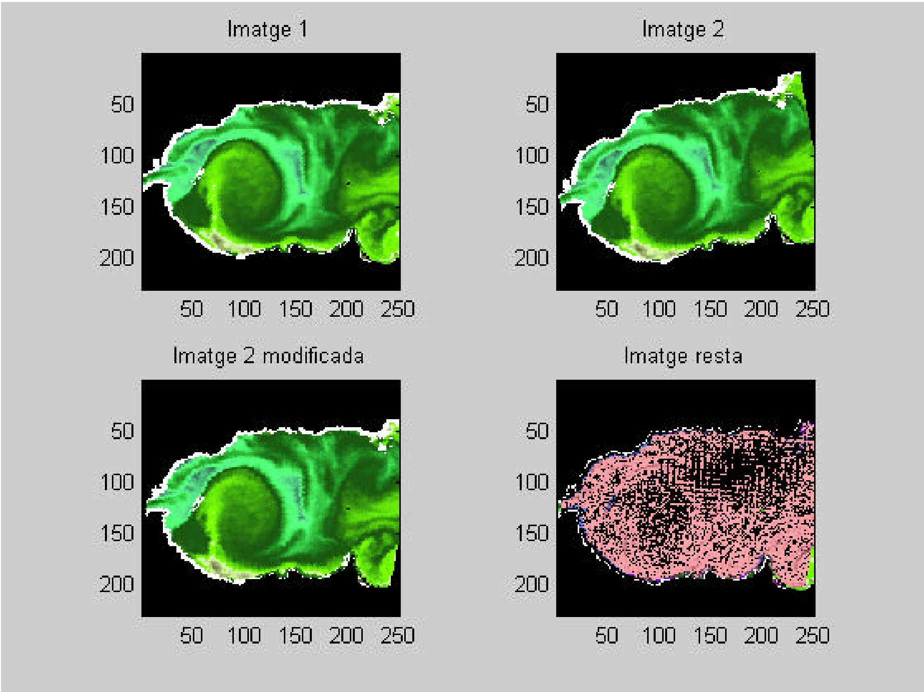


figure 3.4 *MCC with multiscale analysis, version 1. Registration of complete images,* ***Image_2_mod*** *is obtained from* ***Image_2****,* ***Image_1*** *is the ideal transformation result. The difference image is defined as* ***|Image_1 – Image_2_mod|***

MCC with multiscale analysis, version 2

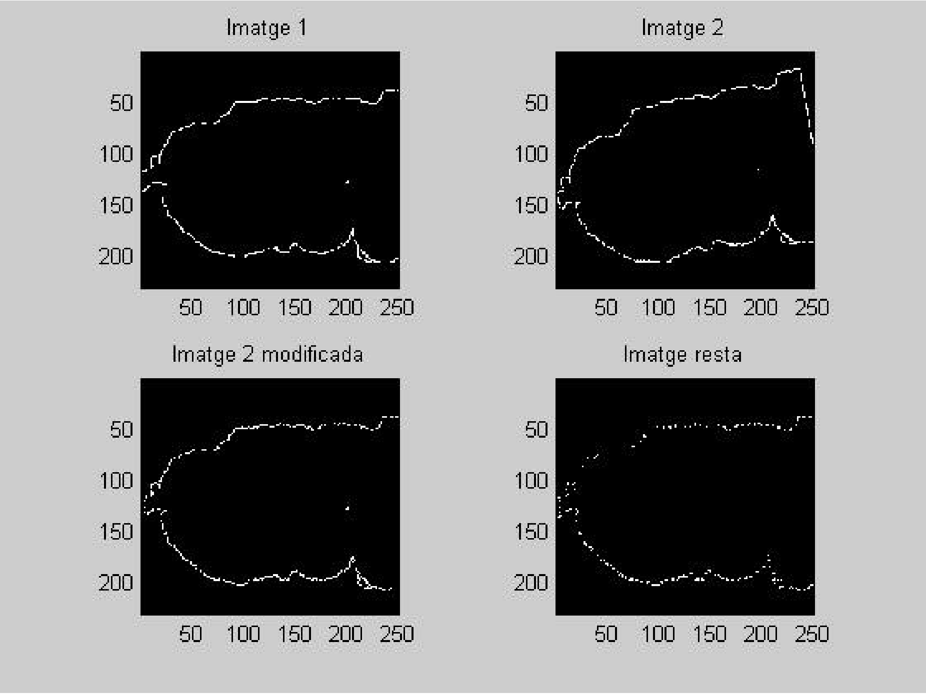


figure 3.5 *MCC with multiscale analysis, version 2. Registration of coastline images.* ***Image_2_mod*** *from* ***Image_2 Image_1*** *is the ideal transformation result. The difference image is defined as* ***|Image_1 – Image_2_mod|***

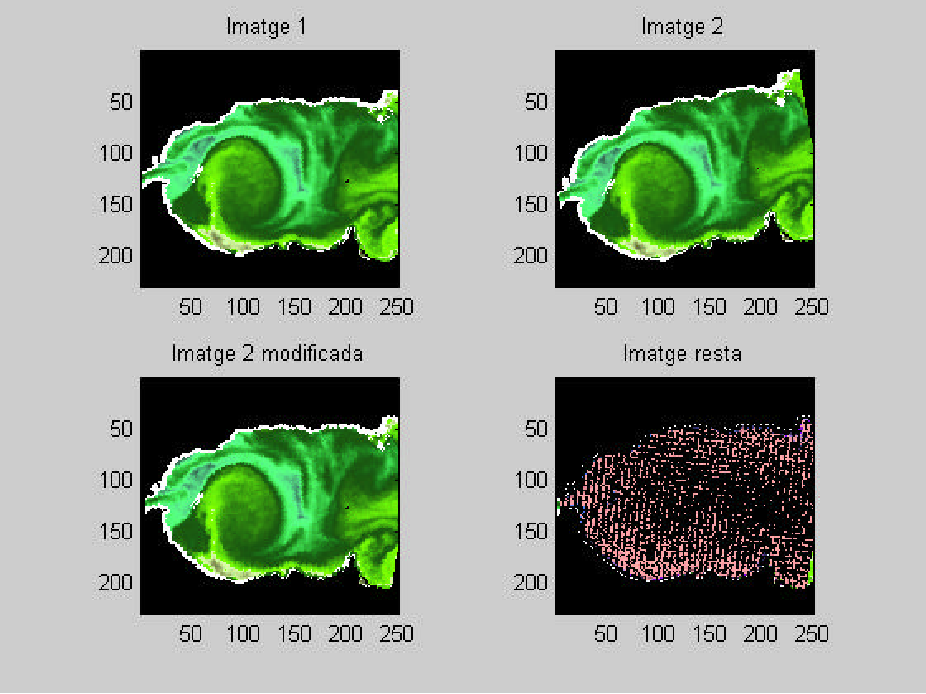


figure 3.6 *MCC with multiscale analysis, version 2. Registration of complete images,* ***Image_2_mod*** *is obtained from* ***Image_2***, ***Image_1*** *is the ideal transformation result. The difference image is defined as* ***|Image_1 – Image_2_mod|***

Direct de-transformation

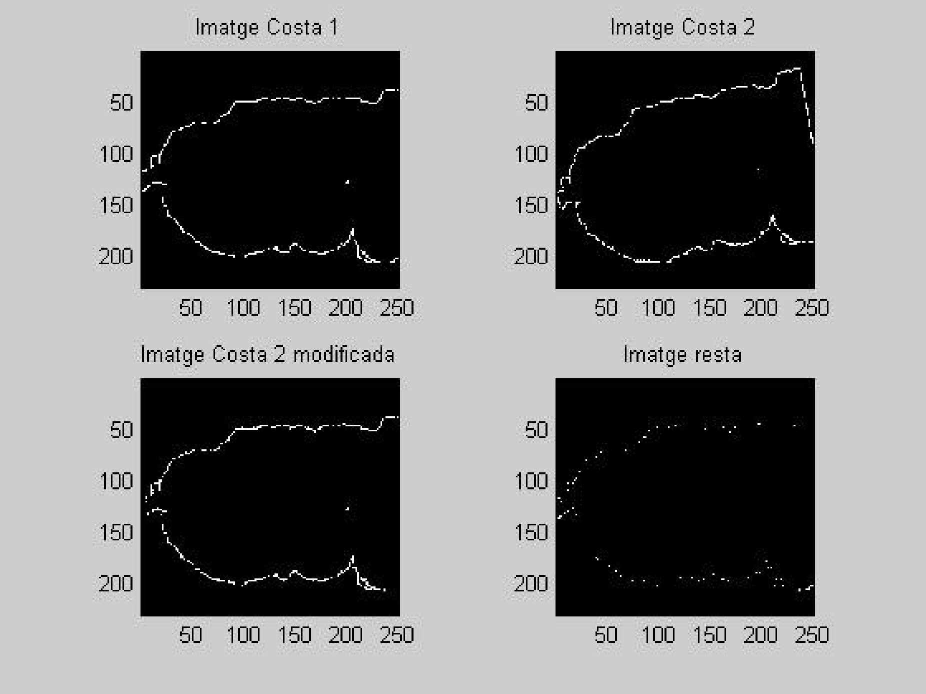


figure 3.7 *Direct de-transformation on coastline images,* ***Image_2_mod*** *is obtained from* ***Image_2****, and* ***Image_1*** *is the ideal transformation result. The difference image is defined as* ***|Image_1 – Image_2_mod|***

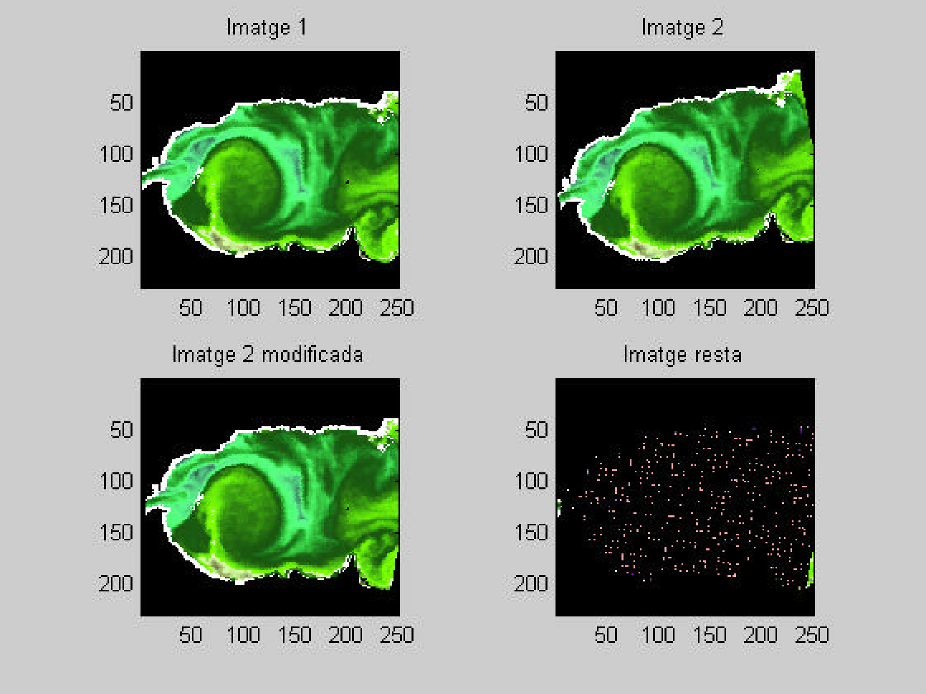


figure 3.8 *Direct de-transformation on complete images,* ***Image_2_mod*** *is obtained from* ***Image_2,*** *and* ***Image_1*** *is the ideal transformation result. The difference image is defined as* ***|Image_1 – Image_2_mod|***

To compare the different methods we can use the previous images. This will be a qualitative comparison, in some cases this will be sufficient.

We want to propose a set of measures derived from difference images, in order to have quantitative comparative measures; a first set from difference images obtained from coastline images, and a second set from difference images obtained from complete images.

The last column is the result obtained with direct de-transformation working with the *nearest neighbour* approximation. It is the maximum ideal value.

a.- From coastline images, difference image **xdif:**

$\mathbf{xdif} = \mathbf{xi1clc} - \mathbf{xi2clcmod}$	Classic	Multiscale 1	Multiscale 2	Direct
$\sum\sum \mathbf{xdif}$	386070	304470	154530	40290
$\mathrm{npunts}_{\mathrm{xdif}} = \dfrac{\sum\sum \mathbf{xdif}}{255}$	1514	1194	606	158
$ratio = \dfrac{npunts_{xdif}}{npunts_{contorn}(836)}$	1.81	1.43	0.725	0.19

$\mathbf{xdif} = \mathbf{xi1clc} - \mathbf{xi2clcmod}$	Multiscale all GCP	Multiscale random GCP
$\sum\sum \mathbf{xdif}$	373830	397290
$\mathrm{npunts}_{\mathrm{xdif}} = \dfrac{\sum\sum \mathbf{xdif}}{255}$	1466	1558
$ratio = \dfrac{npunts_{xdif}}{npunts_{contorn}(836)}$	1.75	1.86

b.- From complete images, difference image **xdif:**

$\mathbf{xdif} = \mathbf{xi1} - \mathbf{xi2mod}$	Classic	Multiscale 1	Multiscale 2	Direct
$\mathrm{m}_{\mathrm{xdif}} = \dfrac{\sum\sum \mathbf{xdif}}{n*m}$	10.7	5.8	2.8	1.2

$\mathbf{xdif} = \mathbf{xi1} - \mathbf{xi2mod}$	Multiscale all GCP	Multiscale random GCP
$\mathrm{m}_{\mathrm{xdif}} = \dfrac{\sum\sum \mathbf{xdif}}{n*m}$	9.4	11.5

4 CONCLUSIONS

In the section 2.3 we have compared error measures from the different methods, but this could lead to wrong conclusions. The set of measures given in the previous section, obtained from difference images is better for comparative purposes.

The first set of result tables, in section 3.a), derived from coastline difference images is as follows: the first row is the difference image global sum; in the second row there is the number of final contour points (dividing the previous value in 255); in the third row there is the ratio of contour points per number of contour points in the original image, which is better when lower. Ideally this last value should be 0. In this practical case, the reference value is 0.19 (from direct de-transformation). A value lower than 1 means that some contour points are superimposed, the closer to 0 we get the more contour points are superimposed. This can be seen in figures 3.1, 3.3 and 3.5, the reference should be figure 3.7. The best result is without a double contour, with only a discontinuous contour line on the difference image. It corresponds to the second version of the multiscale analysis.

Section 3.b) result tables show the mean gray-level value of difference complete images. This should be a zero in an ideal case. In this case it has a practical value of 1.2. This can be observed in figures 3.2, 3.4 and 3.6, figure 3.8 is the reference practical value. As we have a blacker image (closer to zero) we have a better result. As the mean level increases, there are more different points.

We have result tables for other variants: multiscale without GCP selection, where we can see an improvement from classical MCC, just with the multiscale analysis; the last variant is multiscale with random GCP selection, where we can see that GCP selection is a critical aspect for multiscale cases.

As a conclusion the second variant of the MCC using the multiscale analysis method gives the better results, and direct de-transformation gives maximum practical values. The limits on improvement will be stated on further research.

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